INTRODUCTION TO MULTIDIMENSIONAL COSMOLOGICAL MODELS

Paris/Meudon 1998-2001

ELEMENTARY INTRODUCTION

Planck mass, length and time in 4 dimensions

3 fundamental constants : $G = 6,67 \times 10^{-11} \frac{m^3}{kg.s^2}$, $c = 3 \times 10^8 \frac{m}{s}$, $\hbar = 1.06 \times 10^{-34} \frac{kg.m^2}{s}$

Natural (Planck) units are **unique** combinations of these fundamental constants :

- **Planck length** $l_P = \sqrt{\frac{G\hbar}{c^3}} = 1.62 \times 10^{-33} cm$
- **Planck time** $t_P = \sqrt{\frac{\hbar G}{c^5}} = 5.4 \times 10^{-44} s$
- **Planck mass** $M_4 = \sqrt{\frac{\hbar c}{G}} = 2.18 \times 10^{-5} g$

Scales at which quantum gravity effects should be important :

- at distances $\sim l_P$, i.e. at very high energies $M_4 c^2$ (since $l_P \sim (M_4)^{-1}$)
  - *experimentally inaccessible*
- at times $\sim t_P$, i.e. the universe at very early times

Note that

- $M_4^2 \sim \frac{1}{G_4}$, *Planck mass* gives strength of gravity
- $M_4 \gg m_p = 1.67 \times 10^{-24} g$, ~ 19 orders $\rightarrow$ *hierarchy* problem

Energy scales and their hierarchy in 4 dimensional world

$$
\begin{aligned}
m_e &= 5.1 \times 10^{-4}\ GeV = 4.0 \times 10^{-11} cm \\
m_\mu &= 1.0 \times 10^{-1}\ GeV \\
m_\tau &= 1.8\ GeV \\
\\
m_{Higgs} &= 246\ GeV = 8.0 \times 10^{-17} cm \text{ (EW symmetry breaking scale)} \\
m_{LHC} &= 1\ TeV = 2.0 \times 10^{-17} cm \\
\\
M_4 &= 1.2 \times 10^{19}\ GeV = 1.6 \times 10^{-33} cm \text{ (Planck scale (huge))}
\end{aligned}
$$

Hierarchy problem : why there is 16 orders of magnitude between EW scale and Planck scale

- Since $G_4 = M_4^{-2}$ why is gravity so *weak*?
- Since M_4 is a "natural" mass scale why masses of elementary particles are so *small*

D-dim gravity : possible solution?

Classical gravity in D dimensions

Newtonian gravity in D spacetime dimensions is given by *postulating* the Poisson equation as valid in all dimensions

$$\Delta\phi_D = 4\pi G_D \cdot \rho$$

Since $\Delta(\frac{1}{r^{(D-3)}}) = 4\pi\delta_{(D-1)}$, gravitational potential of a point mass is for $D \geq 4$

$$\phi_D \sim \frac{1}{r^{(D-3)}}$$

- $D = 4 \longrightarrow \phi_4 \sim \frac{1}{r}$ and $F_4 \sim \frac{1}{r^2}$
- $D = 5 \longrightarrow \phi_5 \sim \frac{1}{r^2}$ and $F_5 \sim \frac{1}{r^3}$
- $D = 6 \longrightarrow \phi_6 \sim \frac{1}{r^3}$ and $F_6 \sim \frac{1}{r^4}$
- etc.

Notes

- In D-dimensional theory **Planck length**, **time** and **mass** are based on G_D, $\hbar$, c
- $l_{P_D}^{(D-2)} = \frac{\hbar \cdot G_D}{c^3}$
- $[G_D] = L^{(D-4)} \cdot [G_4]$ (since $[\Delta\phi_D]$ is the same in all dimensions)

Compactification and effective theories

Kaluza-Klein (1921-26) $\mathbf{5 = 4 + 1}$

String theories are defined in $\mathbf{D = 4 + n}$ where

- 4 **large** dimensions - observed world
- n extra dimensions - **compactified** and **small** at Planck scale $\sim 10^{-33} cm$
- as of 1996, $D = 10 \rightarrow n = 6$

Our world is truly higher-dimensional with **fundamental mass scale** M_D

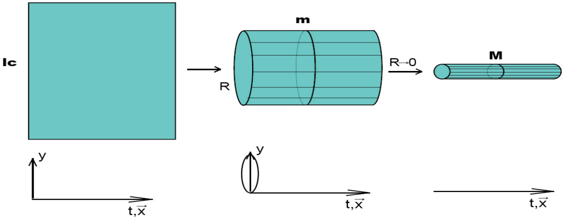
M_4 is its effective value - in 4-dimensional low energy **effective theory** (describing world at distance scales $l \gg l_P$)

Transition between two effective theories by given **renormalization group** method

- from $S = const$ transition $\mathcal{L}_{High} \longrightarrow \mathcal{L}_{Low}$
- $\boxed{\frac{\hbar}{c} \cdot M_4^2 = M_{n+4}^{n+2} \cdot V_n}$, V_n is **extra volume**

Elementary illustration

Consider classical 5 dimensional Kaluza-Klein model with $\mathbf{n} = \mathbf{1}$ extra dimension compactified at circle



m is linear mass density of the ring, **M** is total mass of the ring $\rightarrow M = 2\pi R \cdot m$

$$\begin{aligned} \rho_5(x^1, x^2, x^3, \not{y}) &= m\delta(x^1)\delta(x^2)\delta(x^3) \\ \rho_4(x^1, x^2, x^3) &= M\delta(x^1)\delta(x^2)\delta(x^3) \end{aligned}$$

Since $\rho_5 = \frac{\rho_4}{2\pi R}$ and $\phi_5(x^1, x^2, x^3, \not{y}) = \phi_4(x^1, x^2, x^3)$ (cylindrical symmetry)

$$\Delta_4 \phi_4(x^1, x^2, x^3) = \Delta_5 \phi_5(x^1, x^2, x^3, \not{y}) = 4\pi G_5 \rho_5 = 4\pi \frac{G_5}{2\pi R} \rho_4$$

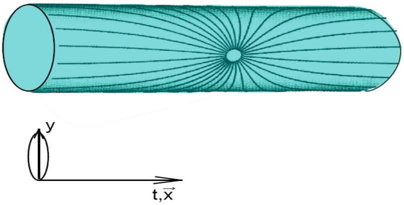
Since $\frac{G_5}{G_4} = 2\pi \cdot R \equiv \mathbf{l_C}$, $\mathbf{l_C}$ ... **compactification length**

- $\frac{\hbar}{c} M_4^2 = M_5^3 \cdot l_C$

In $D = 4 + n$ dimensions $\frac{G_{n+4}}{G_4} = l_C^n$:

- $\boxed{\frac{\hbar}{c} M_4^2 = M_{n+4}^{n+2} \cdot l_C^n}$
- $\boxed{l_{P_{n+4}}^{\,n+2} = l_P^2 \cdot l_C^n}$

Point-like mass



- for $r \ll l_C$: $\mathbf{4+n}$-dimensional gravity with G_{4+n}
- for $r \gg l_C$: $\mathbf{4}$-dimensional effective gravity with $G_4 = \frac{G_{4+n}}{l_C^n}$

Kaluza-Klein particles

How to detect extra dimensions?

1. Classical level : measuring deviation from classical force laws $\sim r^{-2}$:
   - $l_C < 10^{-15} cm$ electromagnetism
   - $l_C < 0.1\ mm!$ gravity
2. Quantum level : extra dimensions can be detected through **Kaluza-Klein particles**

Example : 5-dimensional scalar field $\mathcal{L}_5 = -\frac{1}{2}(\partial_A \Phi)(\partial^A \Phi) \rightarrow \mathcal{L}_4,\ \ A = 0 ... 4$

- $\Phi(x^\mu, y) = \Phi(x^\mu, y + 2\pi R)$
- $\Phi(x^\mu, y) = \sum_{n=-\infty}^{\infty} \phi_n(x^\mu) e^{\frac{iny}{R}}, \qquad (\phi_n^* = \phi_{-n})$

$$\mathcal{L}_5 = -\frac{1}{2} \sum_{n,m=-\infty}^{\infty} [(\partial_\mu \phi_n)(\partial^\mu \phi_m) - \frac{n \cdot m}{R^2} \phi_m \phi_n] e^{\frac{i(n+m)y}{R}}$$

Kaluza-Klein particles

$\boxed{\text{Transition } \mathcal{L}_5 \longrightarrow \mathcal{L}_4 \quad (\text{for scales} > 2\pi R = l_C):}$

Since $S = \int dx^5 \mathcal{L}_5 = \int dx^4 \mathcal{L}_4$:

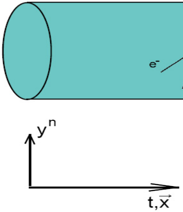
$$\mathcal{L}_4 = \int_0^{2\pi R} dy \mathcal{L}_5 = -\frac{2\pi R}{2} \sum_{n=-\infty}^{\infty} [(\partial_\mu \phi_n)(\partial^\mu \phi_n^*) + \frac{n^2}{R^2} \phi_n \phi_n^*]$$

Normalisation $\varphi_n = \sqrt{2\pi R}\phi_n$:

$$\mathcal{L}_4 = -\frac{1}{2}(\partial_\mu \varphi_0)(\partial_\mu \varphi_0) - \sum_{n=1}^{\infty} [(\partial_\mu \varphi_n)(\partial^\mu \varphi_n^*) + \frac{n^2}{R^2} \phi_n \phi_n^*]$$

- **zero mode** φ_0
- Spectrum of **KK massive modes** φ_n with masses $\mathbf{m_n} = \frac{\hbar}{c} \frac{2\pi n}{l_C}$
- Similarly for electromagnetism A_μ and gravity $h_{\mu\nu}$: $\mathbf{m_n} = \frac{n}{V_C}$

D-dim Kaluza-Klein flat model - SUMMARY

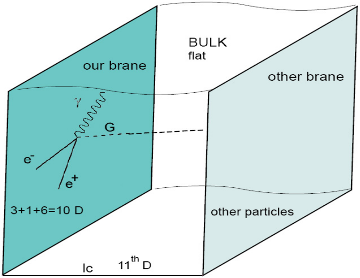


1. $D = 4 + n$, "n" flat dimensions compactified to circle
2. All interactions are truly D-dimensional
3. At low energies $\boxed{r > l_C}$ all interactions effectively 4-dimensional with spectrum of KK particles with $\boxed{m_k \sim \frac{k}{V_n}}$
4. Today's estimation on l_C : no KK particles observed on $100\,GeV$, i.e. $l_C < 10^{-15} cm, \quad n = 1$
5. In string theories $D = 10, \quad l_C$ at Planck scale

BRANE MODELS WITH LARGE EXTRA DIMENSIONS

Branes and Hořava-Witten mechanism (1996)

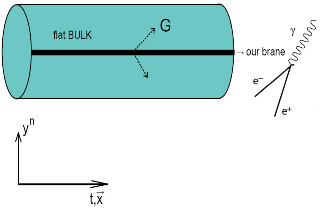
- **Branes** in string theory, Polchinski (1995)
- M-theory in **11** dimensions, Hořava-Witten (1996)
    - 10-branes as edges of 11-dimensional **bulk**
    - *Gravity* propagates in the bulk
    - *Standard Model* **confined** to the brane
    - 6 space dimensions compacified and **small**, 11*th* not necessary



- Effectively **4**-dimensional SM, **5**-dimensional gravity

ADD flat brane model (1998)

- Arkani-Hamed, Dimopoulos, Dvali (1998)
- Model a la Kaluza-Klein + **SM on a brane**

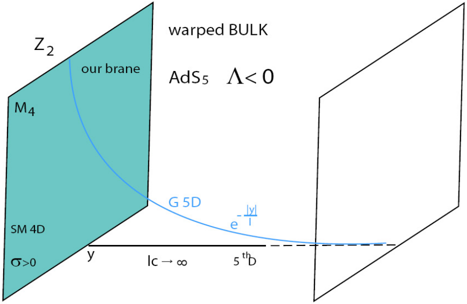


- gravity experimentally proven at $\sim 0.1\,mm$, (32 orders from Planck scale!)
- absence of Standard Model KK particles
- **Hierarchy problem** :
  - if $M_4^2 = M_{n+4}^{n+2} \cdot l_C^n$, $\quad M_{n+4} = m_{Higgs} \sim 1\,TeV$, then $l_C = 10^{32/n-17}cm$
  - if $D = 5 \;\; l_C = 10^{15}cm$
  - if $D = 6 \;\; l_C = 0.1\,mm$
  - **Note** : if SM is $n+4$ dimensional $l_C < 10^{-15}cm, \qquad D \geq 20$

Hierarchy problem can be reformulated in terms of **large extra dimensions** $\sim 0.1\,mm$

Randall-Sundrum warped brane models (1999)

- *RSI* addresses hierarchy problem
- *RSII* shows that there can be **infinite extra dimensions** and not observed
  - Extra dimension is **warped**



Metric in Gaussian coordinates :

- $$\boxed{ds^2 = e^{-\frac{|y|}{l}} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2}$$

- **Minkowski brane** is an edge of the AdS_5 bulk : Z_2**-symmetry** $y \to -y$

Randall-Sundrum warped brane models (1999)

Einstein equations

- $G^{(5)}_{AB} = \kappa_5 \, T^{(5)}_{AB} = -\Lambda \, g^{(5)}_{AB} + diag(\sigma, -\sigma, -\sigma, -\sigma, 0) \, \delta^{(4)}(y)$
- $\mathbf{Z_2}$-symmetry $y \to -y$

Three parameters : brane tension $\sigma > 0$, $\Lambda < 0$, l are not independent :

- $\boldsymbol{\Lambda = -\frac{\kappa_5^2}{6} \, \sigma^2} = -\frac{6}{l^2}$

Potential of a small point mass **m** on the brane in **effective** 4-dimensional theory

1. $V(r) \sim h_{00} = \frac{2mG_4}{r}(1 + \frac{2l^2}{3r^2})$, for $r \gg l$
2. $M_4^2 = M_5^3 \cdot l$, l is **effective compactification length**
3. $l < 0.1mm$

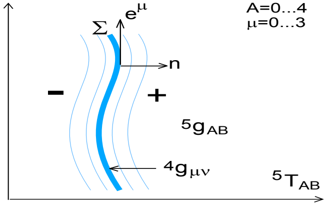
Models with large extra dimensions : SUMMARY

1. **Gravity is D-dimensional**, fundamental scale is M_D
2. **Standard model 4-dimensional**
3. Possibility to solve hierarchy problem by **large extra dimensions**
4. Large extra dimensions
   - **flat** (ADD) $\boxed{l_C \sim 0.1\,mm}$
   - **warped** (RS) $\boxed{l_C \to \infty}$
   - "next door", experimentally accessible
     1. measuring classical force law
     2. searching for KK particles (LHC)
5. **Brane cosmologies** are cosmological generalisations of RS model
6. Effectively General Relativity :
   Our world is a 4-dimensional surface in a 5-dimensional bulk

BRANE-WORLD COSMOLOGICAL MODELS

Thin shell formalism in General Relativity

Israel (1967)



Gaussian coordinates :

- $g^{(5)}_{y\nu} = 0\,;\; g^{(5)}_{yy} = (n.n)\,;\; g^{(5)}_{\mu\nu} = g^{(4)}_{\mu\nu}\,;\; n_\alpha = \delta^y_\alpha$
- $\boxed{K_{\mu\nu} = -\frac{1}{2}\partial_y(g^{(4)}_{\mu\nu})}$

"4 + 1" decomposition of Einstein tensor (Gauss-Codazzi)

- $^{(5)}G^\mu{}_\nu = {}^{(4)}G^\mu{}_\nu + \frac{\partial}{\partial_y}(K^\mu{}_\nu - \delta^\mu{}_\nu\, K) - K\, K^\mu{}_\nu + \frac{1}{2}\delta^\mu{}_\nu(Tr(K^2) - K^2); \quad K \equiv K^\alpha{}_\alpha$

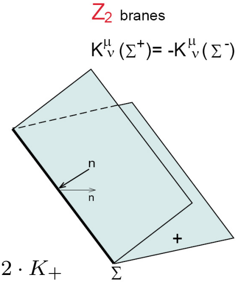
Thin shell formalism in General Relativity

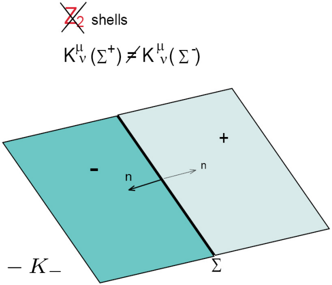
Einstein equations

- ${}^{(5)}G^{\mu}_{\ \nu} = {}^{(4)}G^{\mu}_{\ \nu} + \frac{\partial}{\partial_y}(K^{\mu}_{\ \nu} - \delta^{\mu}_{\ \nu}\, K) + \ldots = \kappa_5\, {}^{(5)}T^{\mu}_{\ \nu},\ /\ \lim_{\epsilon\to 0}\int_{-\epsilon}^{\epsilon} d\,y$
- $\tau^{\mu}_{\ \nu} = \lim_{\epsilon\to 0}[\int_{-\epsilon}^{\epsilon}\, {}^{(5)}T^{\mu}_{\ \nu} d\,y]$

Israel junction conditions

- $\boxed{[K^{\mu}_{\ \nu} - \delta^{\mu}_{\ \nu}\, K]^{+}_{-} = \kappa_5\, \tau^{\mu}_{\ \nu}}$





- $\tau \sim 2 \cdot K_+$

$\tau \sim K_+ - K_-$

Z_2 Symmetric (Brane) Models

Brane cosmologies

- generalisation of RS model adding **cosmological evolution of the brane**
- $\mathcal{M}_\mathbf{4}$ brane $\rightarrow$ **FRW**

Einstein equations

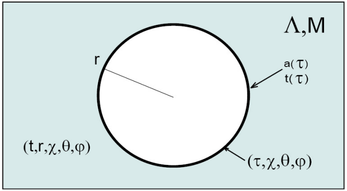
- $G_{AB}^{(5)} = \kappa_5 \, T_{AB}^{(5)} = -\mathbf{\Lambda} \, g_{AB}^{(5)} + diag(\sigma + \rho(\mathbf{t}), -\sigma + \mathbf{p}(\mathbf{t}), -\sigma + \mathbf{p}(\mathbf{t}), -\sigma + \mathbf{p}(\mathbf{t}), 0) \, \delta^{(4)}(y)$
- $\mathbf{Z_2}$-symmetry $y \rightarrow -y$

5-dimensional solution with FRW-slicing imposed is necessary
Schwarzschild-anti-de-Sitter

- $ds^2 = -F(r) \, dt^2 + \frac{dr^2}{F(r)} + r^2 [d\chi^2 + f_k^2(\chi)(d\theta^2 + \sin^2\theta \, d\phi^2)]$
- $\boxed{F(r) \equiv k - \frac{\Lambda}{6} r^2 - \frac{M}{r^2}}$
- brane - spherically symmetric surface *moving* at $\mathbf{r} = \mathbf{a}(\tau)$

Z_2 Symmetric (Brane) Models

- **Bulk solution** $S - AdS_5$: $\mathbf{M}, \mathbf{\Lambda}$



Matching the metric on the brane $\mathbf{r} = \mathbf{a}(\tau)$, $\mathbf{t} = \mathbf{t}(\tau)$:

- $ds^2_\Sigma = -d\tau^2[F(a)\dot{t}^2 - F^{-1}(a)\dot{a}^2] + a^2 d\Sigma^2_k$

FRW on the brane, τ is proper time

- $\boxed{ds^2_\Sigma = -d\tau^2 + a^2 d\Sigma^2_k}$
- $\dot{t} = F^{-1}(a)\sqrt{F(a) + \dot{a}^2}$

tangent and normal vectors

- $e^A_\tau = (\dot{t}, \dot{a}, 0, 0, 0), \quad e^A_i = \delta^A_i, \quad n_A = (-\dot{a}, \dot{t}, 0, 0, 0)$

Z_2 Symmetric (Brane) Models

extrinsic curvature

- $K_{\mu\nu} = -e^A_{\ \mu} e^B_{\ \nu} \nabla_A n_B$
- $K^\chi_{\ \chi} = K^\theta_{\ \theta} = K^\phi_{\ \phi} = \sqrt{h^2 + \frac{k}{a^2} - \frac{\Lambda}{6} - \frac{M}{a^4}}$

Z_2-symmetric Israel junction conditions

- $2 \cdot [K^\mu_{\ \nu} - \delta^\mu_{\ \nu} K] = \kappa_5 \, \tau^\mu_{\ \nu} = diag(-\sigma - \rho, -\sigma + p, -\sigma + p, -\sigma + p)$

1.Conservation equation

- $\dot\rho + 3h(\rho + p) = 0$

2.Friedmann-like equation

- $\boxed{h^2 + \frac{k}{a^2} = [\frac{\kappa_5^2}{36}\sigma^2 + \frac{\Lambda}{6}] + \frac{\kappa_5^2 \sigma}{18}\rho[1 + \frac{\rho}{2\sigma}] + \frac{M}{a^4}}$

  1. $h^2 + \frac{k}{a^2} \sim (\rho + \sigma)^2$, **unconventional cosmology**
  2. $\alpha \equiv \frac{\kappa_5^2}{36}\sigma^2 + \frac{\Lambda}{6}$, effective cosmological constant. RS type model $\alpha = 0$
  3. $\frac{8\pi G}{3} = \frac{\kappa_5^2 \sigma}{18} = \frac{\kappa_5}{3 \cdot l}$, G is Newton's constant
  4. $\frac{M}{a^4}$ is effective radiation (dark radiation)

Simplest Realistic Brane Cosmology

Simplest explicit cosmological solution for $\mathbf{M = 0}$, $\mathbf{k = 0}$, i.e. bulk is AdS_5

- $h^2 = \alpha + \frac{8\pi G}{3}\rho[1 + \frac{\rho}{2\sigma}]$

Consider an equation of state $p = w\rho$, $w = 0$ (dust), $w = 1/3$ (radiation)

- $\dot{\rho} + 3h(\rho + p) = 0 \Rightarrow \boxed{\rho = \rho_0 a^{-q},\ q \equiv 3(1 + w)}$

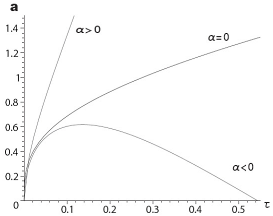
Substituting $X(\tau) = a^q(\tau)$, Friedmann-like equation can be written as

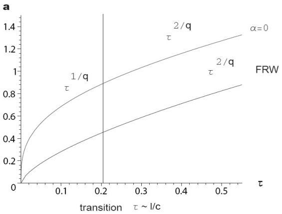
- $\frac{\dot{X}}{q} = \sqrt{\alpha X^2 + \beta X + \xi}, \qquad \beta \equiv \frac{8\pi G\rho_0}{3},\ \xi \equiv \frac{8\pi G\rho_0^2}{6\sigma}$

Solutions

- $a^q = \frac{q^2}{4}\beta\tau^2 + q\sqrt{\xi}\tau \qquad (\alpha = 0)$
- $a^q = \sqrt{\frac{\xi}{\alpha}}\sinh\left(q\sqrt{\alpha}\tau\right) + \frac{\beta}{2\alpha}\left[\cosh\left(q\sqrt{\alpha}\tau\right) - 1\right] \qquad (\alpha > 0)$
- $a^q = \sqrt{\frac{\xi}{-\alpha}}\sin\left(q\sqrt{\alpha}\tau\right) + \frac{\beta}{2\alpha}\left[\cos\left(q\sqrt{\alpha}\tau\right) - 1\right] \qquad (\alpha < 0)$

Simplest Realistic Brane Cosmology





Case $\alpha = 0$

Solution in terms of effective compactification length of AdS_5, $\Lambda = -\frac{6}{l^2}$

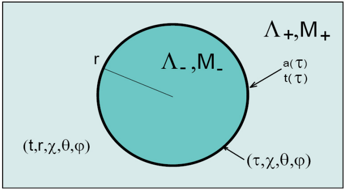
- $a(\tau) \propto \tau^{\frac{1}{q}} (1 + \frac{q}{2l}\tau)^{\frac{1}{q}}$

Asymptotics

1. around big bang $a \sim \tau^{1/q}$
2. late times $a \sim \tau^{2/q}$, $k = 0$ FRW behaviour
3. **transition time** $\sim l/c$, constraints
   - experiment $l < 0.1\, mm \rightarrow \boxed{\tau < 10^{-13} s}$!, $M_5 > 10^8 GeV$
   - nucleosynthesis $\tau \sim 100\, s$, OK

Z_2 Asymmetric (Shell) Models

- **Inner bulk solution** $S-AdS_5$: $\mathbf{M}_-, \mathbf{L}_- \equiv \frac{\mathbf{\Lambda}_-}{\mathbf{6}}$
- **Inner bulk solution** $S-AdS_5$: $\mathbf{M}_+, \mathbf{L}_+ \equiv \frac{\mathbf{\Lambda}_+}{\mathbf{6}}$



extrinsic curvature

- $K^{\pm}_{\mu\nu} = -[e^A_{\ \mu} e^B_{\ \nu} \nabla_A n_B]^{\pm}$
- $K^{+\ \chi}_{\ \chi} = K^{+\ \theta}_{\ \theta} = K^{+\ \phi}_{\ \phi} = \sqrt{h^2 + \frac{k}{a^2} - L_+ - \frac{M_+}{a^4}}$
- $K^{-\ \chi}_{\ \chi} = K^{-\ \theta}_{\ \theta} = K^{-\ \phi}_{\ \phi} = \sqrt{h^2 + \frac{k}{a^2} - L_- - \frac{M_-}{a^4}}$

Asymmetric Israel junction conditions

- $[K^{\mu}_{\ \nu} - \delta^{\mu}_{\ \nu} K]^+_- = \kappa_5 \tau^{\mu}_{\ \nu} = diag(-\sigma-\rho, -\sigma+p, -\sigma+p, -\sigma+p)$

Z_2 Asymmetric (Shell) Models

1.Conservation equation

- $\dot{\rho} + 3h(\rho + p) = 0$

2.Friedmann-like equation

- $\boxed{h^2 + \frac{k}{a^2} = \frac{\kappa_5^2(\rho+\sigma)^2}{36} + \frac{L}{2} + \frac{9\hat{L}^2}{4\kappa_5^2(\rho+\sigma)^2} + \frac{1}{a^4}\left[\frac{M}{2} + \frac{9}{2}\frac{\hat{M}\hat{L}}{\kappa_5^2(\rho+\sigma)^2}\right] + \frac{1}{a^8}\left[\frac{9\hat{M}^2}{\kappa_5^2(\rho+\sigma)^2}\right]}$
  - $M \equiv M_+ + M_-$, $\hat{M} \equiv M_+ - M_-$
  - $L \equiv L_+ + L_-$, $\hat{L} \equiv L_+ - L_-$

  - $h^2 + \frac{k}{a^2} \sim (\rho + \sigma)^2 + \frac{1}{(\rho+\sigma)^2}$, **very unconventional cosmology**

  - **?** realistic cosmology - FRW at late times **?**

Simplest Realistic Shell Cosmology

Simplest explicit cosmological solution for $\mathbf{M = 0}$, $\mathbf{k = 0}$, i.e. bulk is AdS_5

- $h^2 = \frac{\kappa_5^2(\rho+\sigma)^2}{36} + \frac{L}{2} + \frac{9\hat{L}^2}{4\kappa_5^2(\rho+\sigma)^2}$

recovering standard cosmology at late times

- $L_- < 0, L_+ < 0;$ choice $L_- < L_+$
- fine-tuning of σ :
  1. $\sigma_+ = \frac{3}{\kappa_5^2}(\sqrt{-L_-} + \sqrt{-L_+}) > 0$
  2. $\sigma_- = -\frac{3}{\kappa_5^2}(\sqrt{-L_-} - \sqrt{-L_+}) < 0$

Friedmann equation at late times, $\rho \to 0$:

- $h^2 = \frac{8\pi G}{3}\rho + O(\rho^2)$
  - $\frac{8\pi G}{3} = \frac{2\kappa_5}{3}\frac{\sqrt{L_+L_-}}{\sqrt{-L_-}\pm\sqrt{-L_+}}$, G is Newton's constant

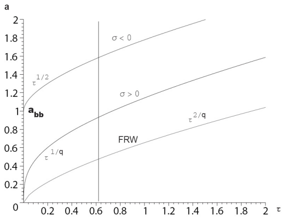
Simplest Realistic Shell Cosmology

Complete scale factor evolution for $p = w\rho$ in parametric form :

- $a(y) = y^{-1/q}$
- $\tau_+(y) = \frac{6}{q\kappa_5|\sigma_+|}\int_y^\infty \frac{(1+z)dz}{z^{3/2}\sqrt{(2+z)(2z+z^2+\gamma_+)}} \qquad y \in (0,\infty)$
- $\tau_-(y) = \frac{6}{q\kappa_5|\sigma_-|}\int_y^1 \frac{(1-z)dz}{z^{3/2}\sqrt{(2-z)(2z-z^2+\gamma_-)}} \qquad y \in (0,1)$
  - where $\gamma_\pm \equiv \frac{36}{\kappa_5^2\sigma_\pm^2}\sqrt{L_+L_-}$

Asymptotic

- big bang $\tau_\pm \to 0$, **1.**: $a \sim \tau_+^{1/q}$, **2.**: $a \sim a_{bb}(1 + \frac{1}{q}\sqrt{\frac{\tau_-}{d}})$, $R \to \infty$
- late times $\tau_\pm \to \infty$, $a \sim \tau_\pm^{2/q}$ FRW



Brane cosmologies : SUMMARY

1. **Large extra dimensions possible** : good motivation in high energy physics

2. Effectively General Relativity : **our world is a** 4**-dimensional surface in a** $4+n$**-dimensional bulk**

3. Cosmologies Z_2-symmetric and asymmetric
   - **FRW et late times**
   - Modifications at very early times (modified primordial cosmology)

4. Today
   - **Theory** : **string phenomenology** : brane inflation, black holes, Einstein-Gauss-Bonnet branes etc.
   - **Experiment** : searching for extra dimensions (LHC)

5. **Brane perturbations** analysis (CBR anisotropies, etc.)