Satellite positioning systems

- Methods of satellite navigation
- Global Positioning System
- Examples of practical usage

Methods of satellite navigation

- Angular measurements
- Doppler tracing
- Distance measurement (via time delay)
  - Active
  - <u>Passive</u>
- Differential (interferometric) methods

Satellite navigation is <u>accurate</u> and <u>global</u>.

Passive systems for distance measurement

$$\sum_{i=1}^{3}\left(x_i^{(s)} - \underline{\underline{u_i}}\right)^2 = \Big(\underbrace{R^{(s)}}_{c\,\Delta t^{(s)}} - \underline{\underline{\tau_{\mathrm{b}}}}\Big)^2$$

$u_i \rightarrow$ user position

$\tau_{\mathrm{b}} \rightarrow$ user clock bias

$x_i^{(s)} \rightarrow$ satellite position $(s = 1,\ldots,4)$

$R^{(s)} \rightarrow$ pseudo range

Simplified version of equations used by actual GPS receivers for position-velocity-time calculations.

Pseudorange

Pseudorange $R^{(s)}$ includes

- Actual distance between satellite and user
- Clock bias
- Atmospheric delay
- Receiver noise
- Relativistic effects

Relativistic effects

Clocks cannot be synchronized for all observers residing on (rotating) Earth: Sagnac effect.

Total daily relativistic effects amount to $\approx 4 \times 10^4$ ns for a GPS satellite clocks relative to ground clocks – satellite clocks are designed with a frequency offset.

Relativistic effects can be combined in two corrections

- The average satellite clock frequency is shifted down by factor 446×10^{-12}. This includes
  - Gravitational shifts of ground and satellite clocks
  - Second-order Doppler shift
- Correction for eccentric orbits
  - Periodically varying; the amplitude is order of 0.2*e* ns

see N. Ashby

Practical realization: NAVSTAR – GPS

- Space segment
- Control segment
- User segment

The satellites transmit signals on two frequencies:

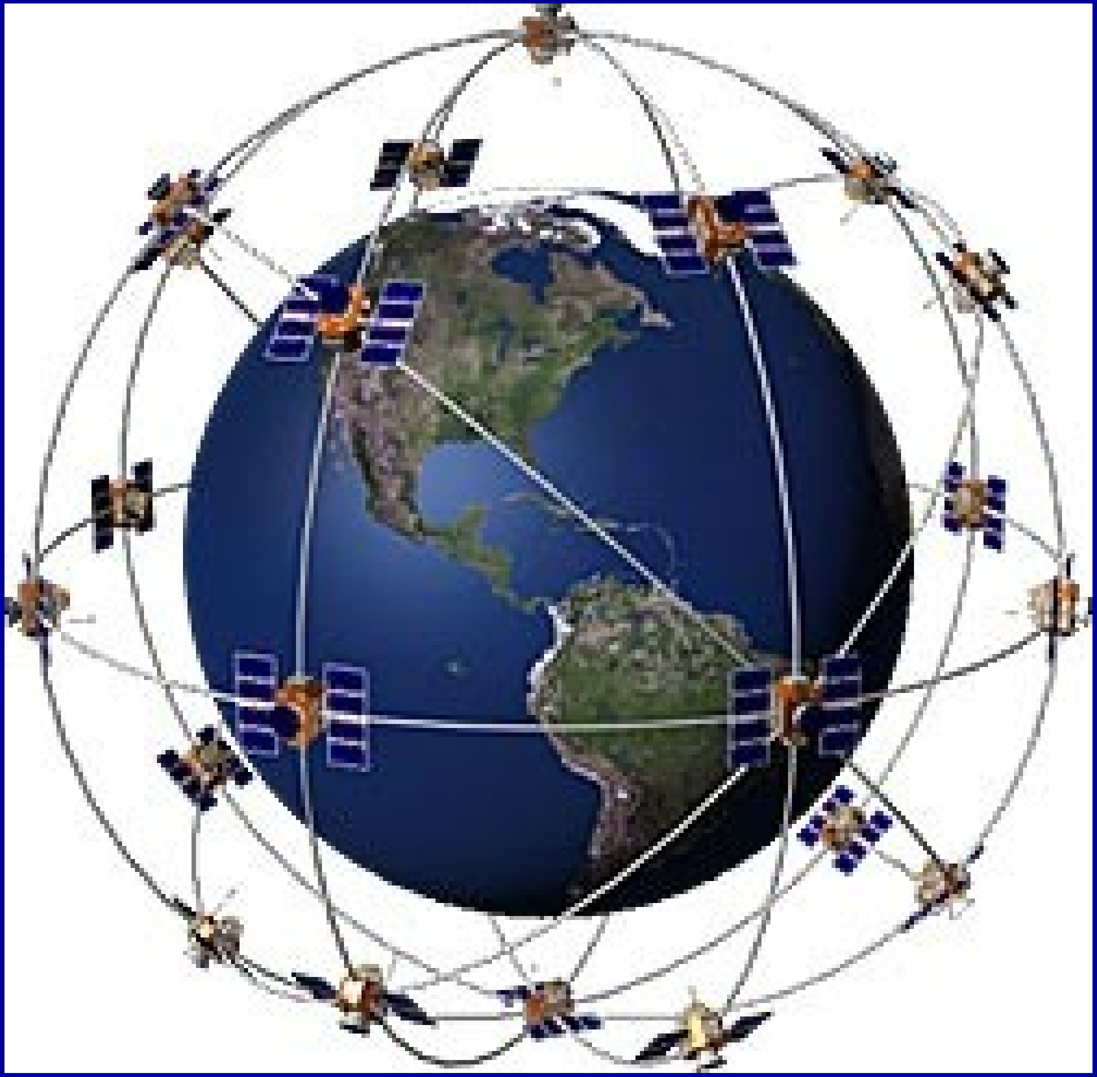
- L1 – 1575.4 MHz
- L2 – 1227.6 MHz

Signal modulation:

- Coarse/acquisition (C/A) code on L1 – 1.023 MHz
- Precision (P/Y) code on L1, L2 – 10.23 MHz

+ Data signal (transmitting satellite ephemeris, satellite constellation almanac, signal propagation correction)

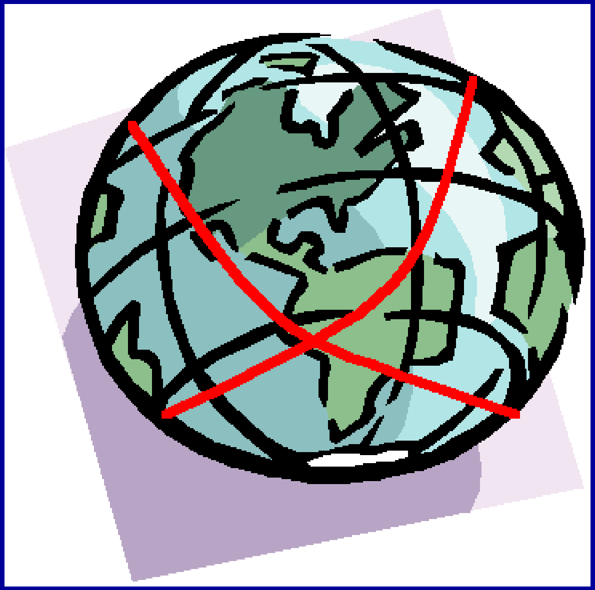
The three segments – 1

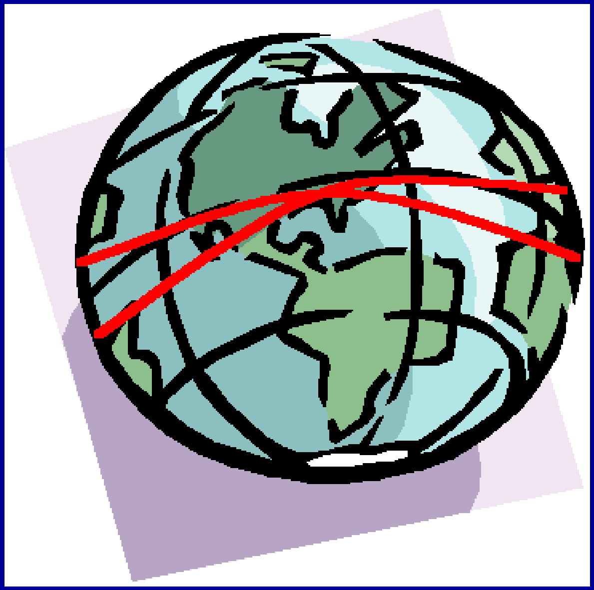


- 24 satellites
- 11 hour 58 min orbit
- 55 deg inclination
- 20 200 km altitude

At least four satellites are always visible above horizon (with appropriate geometric relationship).

Geometric dilution of precision

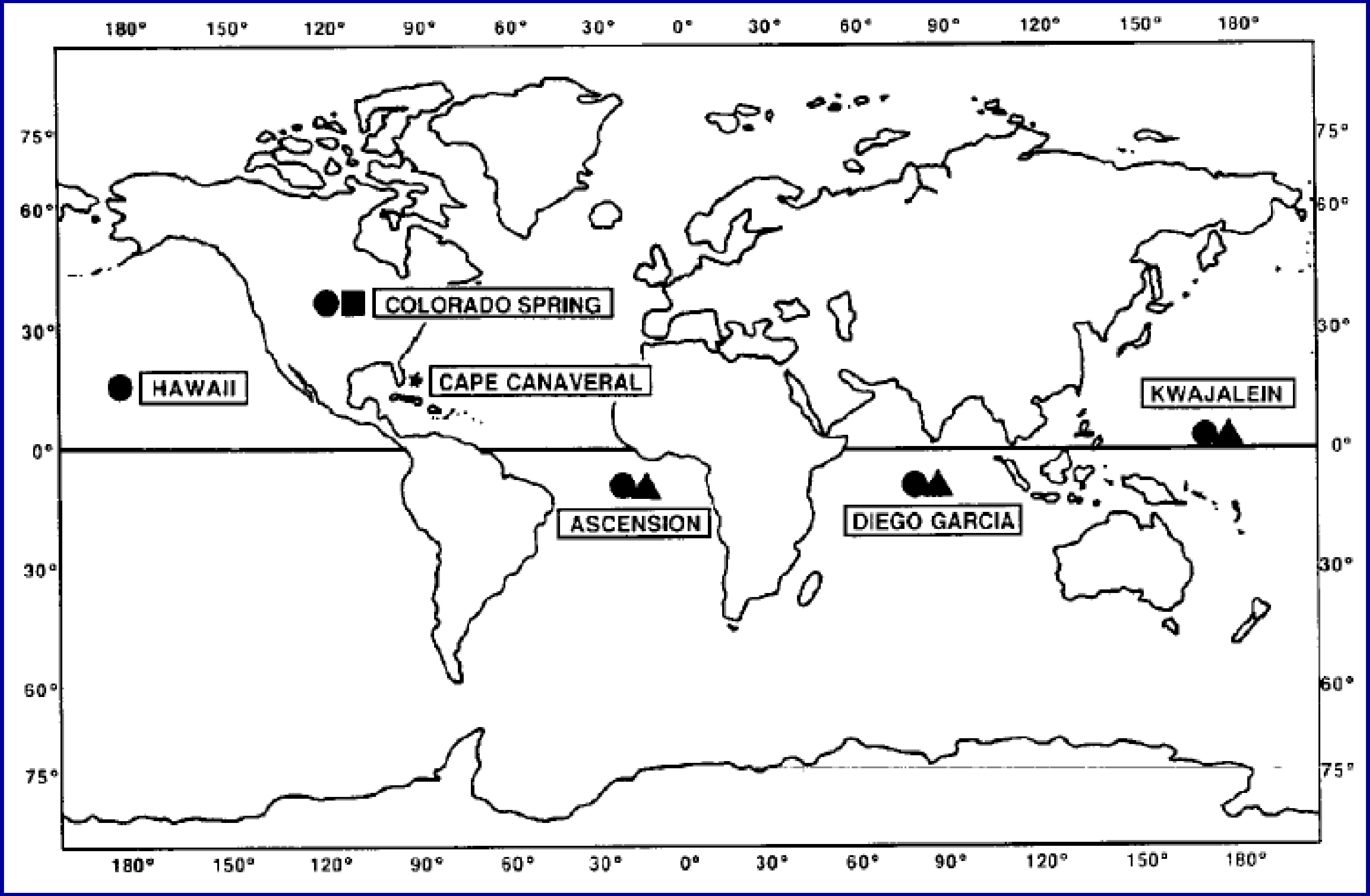




Good geometry versus Bad geometry

Geosynchronous orbits are located above the equator $\Rightarrow$ all in the same plane. GPS satellites orbit in three different planes.

... 2



- Master control station (manages the constellation)
- Monitor station (passive tracking)
- Ground antenna (periodic upload of ephemeris)

... 3

Receiver

- Sequential
- Continuous
- All-in-view
- Differential
- …

Satellite codes

Satellite codes are pseudo-random (PRN) sequences

$$s(t) = D(t)\left(s_C(t) + s_P(t)\right)$$

$$s_C(t) = A_C\, C(t)\sin(2\pi L_1 t)$$

C/A – 1024 bit coarse code; 1 ms period; different PRN for each satellite; not encrypted ("<u>c</u>lear <u>a</u>ccess")

$$s_P(t) = A_{P_1} P(t)\sin(2\pi L_1 t) + A_{P_2} P(t)\sin(2\pi L_2 t)$$

P/Y – precision code; 267 days period (restarts every week); encrypted ("<u>p</u>rotected")

Navigation message is superimposed on both codes.

Satellite selection, signal acquisition

Tracking by a receiver begins by determining, which satellites are visible

- Satellite visibility is based on almanac and the initial receiver estimate, or
- Systematic search of the sky

The satellite signal power is below the receiver thermal noise $\rightarrow$ correlation techniques (the receiver shifts a copy of the code to match the incoming satellite code).

The receiver tracks the satellite by adjusting its internal frequency $\rightarrow$ Doppler measurement.

Improving accuracy

Accuracy can be increased using differential GPS methods

- Local (e.g. pseudolits)
- Wide area (e.g. WAAS)

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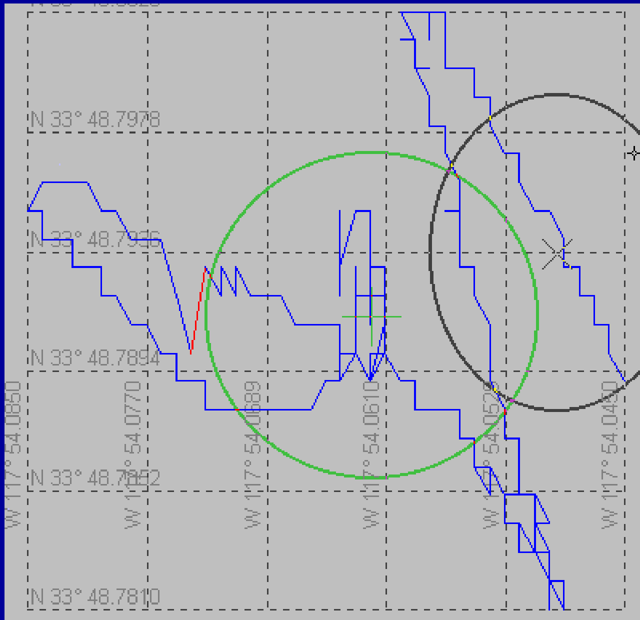
Selective availability – Intentional degradation of C/A code accuracy (can be avoided if cryptographic code is known).

Error budget

Segment	Source of error	Normal mode [m]	Differential mode [m]
Space	Clock stability	6.5	–
	Perturbations	2.0	–
Control	Ephemeris	8.2	–
User (P/Y)	Ionospheric delay	4.5	0 – 4.5
	Tropospheric delay	3.9	0 – 3.9
	Receiver noise	2.9	4.1
	Multipath	2.4	3.4
UERE	$\sqrt{\Sigma\sigma_i^2}$	13.0	5.8 – 8.3

Examples



Visualization of a typical GPS signal

Mapping usage